

Apollo 17 anomalous ascent trajectory

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When humans visited the Moon for the last time, in December of 1972 as a part of the Apollo 17 mission, the ascent of the Lunar Module (LM) from the lunar surface was captured by a remotely-operated camera that was left on the Lunar Roving Vehicle, and broadcast live across television networks worldwide.

From the broadcast recording available on YouTube.com, we first reconstruct the elevation of the ascending LM Ascent Stage (AS) from the camera vantage point. As a part of that reconstruction we find what NASA camera operator Ed Fendell did with the camera to track the LM AS.

We use NASA sources to reconstruct nominal Orbit Insertion Phase of the Apollo 17 LM, and find that its trajectory does not match the television broadcast. We consider two types of vessels to explain the scene. The first type is a rocket with the same propulsion as that of Apollo but with a much quicker Reactive Control System (RCS). We find that depending on turn-rate limits the rocket either completely fails to reproduce the scene as broadcast, or reproduces most of the broadcast but in a trajectory which cannot be performed with Apollo's guidance logic and the RCS. The second type is a roller coaster, comprised of a jet-propelled cart sliding along the tracks pitched so that they mimic the ascent trajectory. We show how a roller coaster can be constructed that matches the broadcast elevation perfectly, and has enough flexibility to address its minute variations.

The failure of rocket to reproduce the scene as broadcast in combination with the roller coaster success, strongly suggests that the television broadcast comprised a staged, scaled-down ascent, and not an actual LM ascent from the lunar surface.

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1. INTRODUCTION

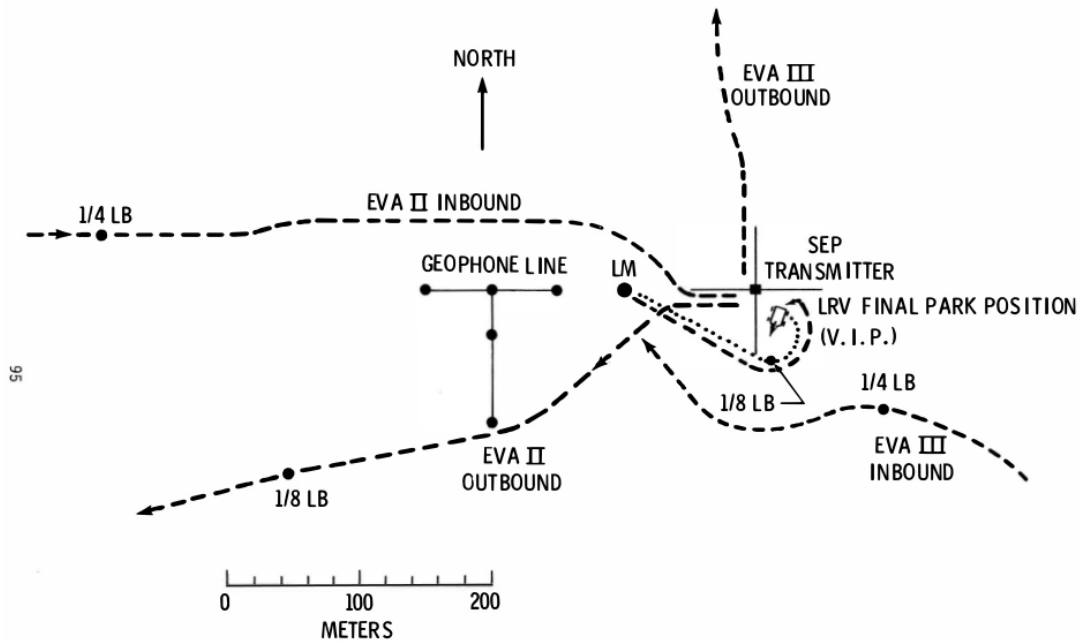


FIG. 1: The map of the Apollo 17 landing site on the Moon [1] shows the location of the Lunar Roving Vehicle (LVR), which carried the Ground Controlled Television Assembly that broadcasted the ascent.

Apollo 17 was the final mission of NASA's Apollo program. It was launched on December 7, 1972, at 12:33 Eastern Standard Time (EST). It spent three days on the lunar surface. For extended mobility it brought a Lunar Roving Vehicle (LRV). The mission landed in the Taurus Littrow valley and completed three moon-walks, taking lunar samples and deploying scientific instruments. The crew returned to Earth on December 19, 1972, after a 12-day mission. [2]

We are interested in the ascent of the Apollo 17 Lunar Module (LM) Ascent Stage (AS) from the lunar surface, which took place on December 9, 1972. The ascent was captured by a camera on the LRV parked nearby, cf. Fig. 1. The TV camera was controlled remotely by NASA camera operator Ed Fendell, who skillfully managed to continuously track the LM AS for the first 29 seconds of ascent. To this day, this is considered to be one of the best video recordings from the entire Apollo program. There are many copies on YouTube.com of which [3] is one of the best quality and, for our purpose, is sufficiently long in duration.

2. ASCENT FROM THE LUNAR SURFACE

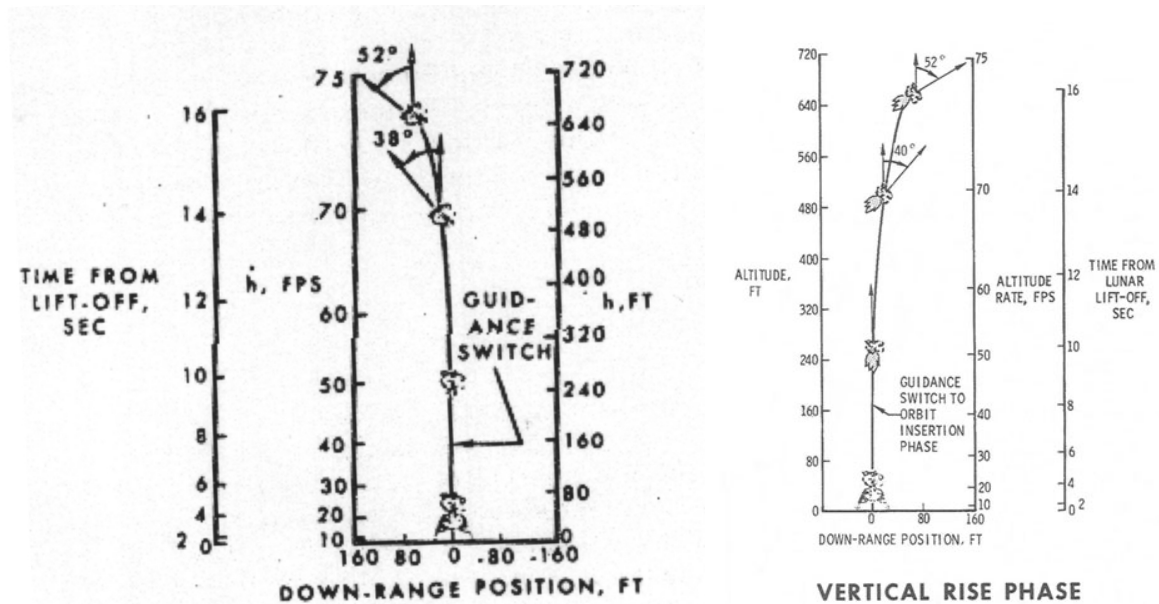


FIG. 2: The vertical rise phase with the nominal thresholds for the guidance logic reported for Apollo 11 ([4], left panel) and posted on the web site discussing the Apollo 17 ascent ([5], right panel). Please note the orientation of down-range position (LM Z-axis) in two panels: the Apollo 11's matches the orientation of the LM in Fig. 6 (Z+ with the doors on the left, Z- direction on the right).

The following is converted from the description of the Apollo 11 lunar ascent. [4] The ascent from lunar surface was the last part of the lunar stay of any Apollo mission. The single objective of the ascent was to achieve an orbit from which rendezvous with the orbiting Command and Service Module (CSM) could be performed. The target orbit was 16.7-by-44.4 km (9-by-24 nm) at a true anomaly of 18° , and the 18.3 km (=60,000 ft) altitude. The time of liftoff was chosen to provide the correct phasing for rendezvous. The target requirements were height, velocity, and orbit plane. Unlike the Descent Propulsion System (DPS), which featured a throttleable gimbal drive capable of adjustments of up to $\pm 6^\circ$, the Ascent Propulsion System (APS) had a fixed gimbal angle and was not throttleable (it possessed only “on” and “off” states). As the vessel was steered by the guidance logic that controlled the Reactive Control System (RCS), the ascent required no input from the astronauts. For that reason, the early phases of ascent did not change between the Apollo

missions.

The ascent was divided into two phases: Vertical Rise (VR), and Orbit Insertion (OI) as illustrated in Fig. 2:

1. The Vertical Rise Phase was required for the LM AS in order to achieve terrain clearance.

We refer to its first two seconds after liftoff as the “Descent Stage Clear,” during which the guidance logic maintained initial attitude. After that, the attitude was pitched to the vertical, while rotating (yawing) the vessel to the desired azimuth so that the LM AS XZ-plane was parallel to the orbital plane of the CSM.

2. The early Orbit Insertion Phase starts with the first pitch adjustment in a sequence of such adjustments, all performed by the guidance logic. In popular vernacular the first pitch adjustment was referred to as the “pitch-over.”

The nominal thresholds for the guidance logic to terminate the Vertical Rise Phase were 10 s in duration, or velocity exceeding 15.3 m/s (=50 fps), see discussion in [4]. According to Fig. 2, the Apollo 11 pitch-over started at 10 s after liftoff, which by 14 s has reached the pitch of 38° , and by 16 s reached 52° . This suggested that the pitch rate commanded by the guidance logic to the Reactive Control System (RCS) was $9^\circ/\text{s}$ for the first 6 s of the early Orbit Insertion.

In the later sections we discuss more precisely what pitch adjustments an Orbit Insertion requires.

3. TRIBUTE TO ED FENDELL

We extract the absolute elevation of the Lunar Module (LM) Ascent Stage (AS) with respect to the camera for the duration of the Vertical Rise and the early Orbit Insertion Phases. In doing so we recover the actions of the NASA camera operator Ed Fendell, who managed to continuously track the LM AS for the first 29 seconds after liftoff.

The camera that recorded this ascent was an RCA J-Series Ground Commanded Color TV Camera. [6, 7] It featured a Ground Commanded Television Assembly (GCTA), which allowed it to zoom, tilt (change attitude) and pan (change azimuth). The tilt rate was reportedly limited to three values [8] $\omega = 0, +/ - 3^\circ/\text{s}$, and the zoom range was from focal length of 12.5 mm and field-of-view (FOV) of 9° , to 75 mm and FOV of 54° , with zoom-ratio 6:1. The camera employed a so called Field-sequential Color System (FSCS) [9], in which single color fields are collected through a rotating red, green and blue filter. As reported on clavius.org, even though the camera was capable of recording images at 60 frames per second (fps), it was modified to broadcast single color fields at 30 fps to reduce bandwidth. On Earth, three consecutive fields would be assembled in a single color frame at the same 30 fps rate.

There are a number of video clips of the Apollo 17 LM ascent on YouTube.com. For this investigation we have chosen [3] which is of reasonably high quality. We therefore deemed it unnecessary to contact the Jet Propulsion Laboratory (JPL) for a further copy of the ascent recording. This video clip is 6 mins 50 secs in duration, its source is in Flash format, has a frame rate of 30 fps, where the frames are 426x240 pixels. The pixel values comprise four channels (RGBA) 8-bit deep, thus in the range of 0 to 255. The frames contain sub-frames of size 320x240 pixels.

We download the video clip directly from YouTube.com web site using the tool `youtube-dl`. [17] We use standard tool `FFmpeg` [10] to extract individual images in JPEG-format at the rate $r = 10$ fps. [18] We extract red channel from each JPEG-image, and create new monochromatic image in which all color channels are identical (to red). The numbering of images allows one to determine their absolute time with respect to the beginning of the video recording as $t_n = (n - 1)/r$, where n is the frame index.

After inspecting the entire video clip [3], the following can be said about the ascent and the camera filming it:

1. The liftoff or zero time is in the frame 1841₁₀ ([3] 03:04.1).
2. The camera starts to zoom out from the frame 1839₁₀ ([3] 03:03.9), at $t = -0.2$ s.
3. The time it takes the camera to zoom in and out is $\Delta t_Z = 13.7$ s. This follows from the frames 3005:3142₁₀ ([3] 05:00.5 to 05:14.2), depicting one such zoom-in.
4. In the first part of the ascent the LM AS is continuously in the camera field-of-view (FOV). This is given in the frames 1841:2128₁₀ ([3] 03:03.1 to 03:32.8). From this sequence of frames we isolate the LM AS crew compartment and record vertical pixel coordinate $Y(t)$ of its lower edge, and horizontal pixel coordinate $X(t)$ of its center. This is illustrated in Fig. 6 in Sec. 4.

We base our reconstruction of absolute elevation on two assumptions:

1. The LM AS elevation, call it $\eta = \eta(t)$, is of the continuity class $C(1)$, meaning that left and right limits of $\dot{\eta} = d\eta/dt$ are equal, $\dot{\eta}^L(t) = \dot{\eta}^R(t), \forall t \geq 0$, respectively. This follows from the equations of motions that describe the trajectory of LM AS.
2. In the absence of panning, the LM AS elevation η and azimuth α can be represented as,

$$\eta = \mu + \Phi \cdot \rho, \quad (3.1)$$

and

$$\alpha = \Phi \cdot \zeta, \quad (3.2)$$

respectively.

Here, $\mu = \mu(t)$ is the camera tilt, that is, the elevation of the center of the camera frame. From the camera specifications it is known that $\dot{\mu}$ is limited to three values: 0, or no tilt, and $\pm 3^\circ/\text{s}$, for camera tilting up or down.

Then, $\Phi = \Phi(t)$ is the camera zoom function, which is given as the ratio of the horizontal or vertical FOV at time t to the maximal or FOV(1:1),

$$\Phi(t) = \frac{\text{HFOV}(t)}{\text{HFOV}(1:1)} = \frac{\text{VFOV}(t)}{\text{VFOV}(1:1)}. \quad (3.3)$$

From the video clip it transpires that when the camera zoom was engaged it went from one extremum to the other and did not stop or change direction mid-way. Considering

that the zoom was driven by an electric motor rotating at a constant rate which direction could be changed, it transpires that the zoom-in (Φ^+) and the zoom-out (Φ^-) functions are reverse of each other. This we write as,

$$\Phi^+(t) = \Phi^-(\Delta t_Z - t), \quad (3.4)$$

where $\Delta t_Z = 13.7$ s is known.

Lastly, $\rho = \rho(t)$ is the relative elevation assuming the maximal vertical FOV of the camera,

$$\rho(t) = \text{VFOV}(1:1) \times \left(\frac{1}{2} - \frac{Y(t) - \frac{1}{2}}{\text{VFRAMESIZE}} \right), \quad (3.5)$$

while $\zeta = \zeta(t)$ is the relative azimuth in the maximal horizontal FOV of the camera,

$$\zeta(t) = \text{HFOV}(1:1) \times \left(\frac{X(t) - \text{HFRAMEOFFSET} - \frac{1}{2}}{\text{HFRAMESIZE}} - \frac{1}{2} \right). \quad (3.6)$$

Here, $X = X(t)$ and $Y = Y(t)$ are the horizontal and vertical positions of the points on the LM AS, which are found by image inspection. From the camera specifications we have $\text{HFOV}(1:1)=54^\circ$, $\text{VFOV}(1:1)=41^\circ (=3/4 \cdot \text{HFOV}(1:1))$, and $\text{VFRAMESIZE}=240$, $\text{HFRAMESIZE}=320$, and $\text{HFRAMEOFFSET}=50$. This allows us to immediately determine ρ and ζ , which we show Fig. 3.

Inspection of ρ in Fig. 3 suggests that $\dot{\rho}(t)$ has a number of jump-discontinuities. We isolate time instances when the jump-discontinuities occur in the first column of Tbl. I. Let t_i be an instance when $\dot{\rho}_i = \dot{\rho}(t_i)$ is discontinuous, meaning that the left $\dot{\rho}_i^L$ and the right $\dot{\rho}_i^R$ limits differ, $\dot{\rho}_i^L \neq \dot{\rho}_i^R$. Because $\dot{\eta}$ is continuous, this implies that every discontinuity in $\dot{\rho}$ is canceled either by discontinuity in $\dot{\mu}$ (camera tilt) or in $\dot{\Phi}$ (zoom transition).

First discontinuity to be identified is the zoom-transition at 13.5 s, marked in Fig. 3 as green triangle. The zoom-out starts at $t_{zo} = -0.2$ s, and is of known duration of $\Delta t_z = 13.7$ s, so it ends at the zoom-transient time of $t_{zT} = t_{zo} + \Delta t_z = 13.5$ s. However, toward the end of the first part of the video clip, it is obvious that past t_{zT} the camera is zooming-in. Obvious conclusion is that the zoom-out continued to 1:1 and then changed to zoom-in which continued to maximal value 6:1 that was reached at 27.2 s. This zooming behavior is listed in Tbl. I.

Then, the jump-discontinuity in $\dot{\rho}$ at $t = 13.5$ s has to be compensated by the jump-discontinuity in $\dot{\Phi}$, so

$$\frac{\dot{\rho}^L - \dot{\rho}^R}{\rho}(t^*) = \frac{\dot{\Phi}^R - \dot{\Phi}^L}{\Phi}(t^*). \quad (3.7)$$

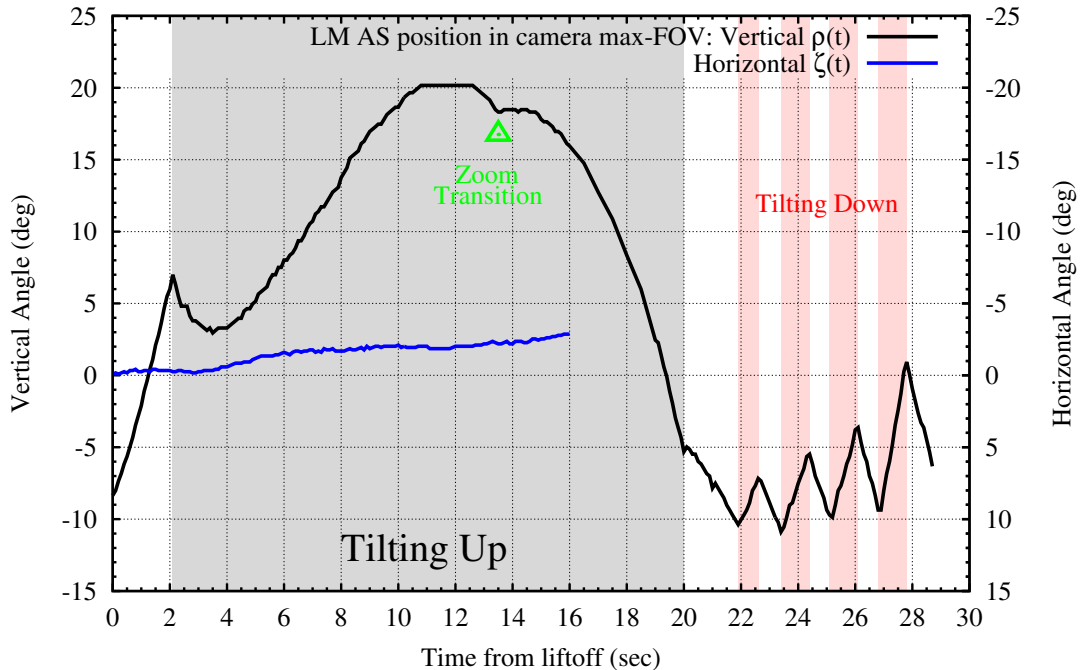


FIG. 3: The relative elevation ρ (in black) and the azimuth ζ (in blue) of the LM AS in the camera FOV(1:1), and the camera movements (tilt up, in gray; tilt down in red; zoom transition at green point) revealed by the jump discontinuities in $\dot{\rho}$.

However, as $\dot{\rho}$ is noisy when calculated point-to-point this relationship is of little use in finding Φ .

The discontinuities in $\dot{\mu}$ are most numerous and most easily identified from the video clip. At 2.1 s the camera starts to tilt upwards to follow the ascent, so $\dot{\mu}$ jumps from 0 to $+3^\circ/\text{s}$. Similarly, a sequence of four downwards tilts in which $\dot{\mu}$ jumps from 0 to $-3^\circ/\text{s}$ starting at 21.9 s is identified as such, because they keep the LM AS moving down in the camera FOV. So between 2.1 s and 21.9 s the camera had to stop tilting upwards, and the discontinuity at 20 s after liftoff is its obvious location. This completes our discontinuity analysis based on $\dot{\rho}$ which we summarize in Tbl. I. In Fig. 3 we show functions η and ζ , and zoom and tilt regions of the camera (gray for camera tilting up, red for camera tilting down, no shade for camera not tilting, and green triangle that indicates transition from zoom-out to zoom-in). What is most important is that from the jump-discontinuities in $\dot{\rho}$ and in $\dot{\mu}$ at times t^* , we can deduce values of the zoom-function as,

$$\Phi(t^*) = \frac{\dot{\mu}^L - \dot{\mu}^R}{\dot{\rho}^R - \dot{\rho}^L}(t^*). \quad (3.8)$$

For better numerical accuracy in determining left and right slopes of $\dot{\rho}$, we can now use neighborhood around discontinuities. In Tbl. III we collect all so computed $\Phi^+(t^*)$ for $t^* \geq t_{ZT}$.

We assume that for the duration of zoom-in $t' = t - t_{ZT} \in [0, \Delta t_Z]$, the zoom-in function can be written as,

$$\Phi^+(t') = \frac{1 - q_1 t'}{1 + q_2 t'}, \quad (3.9)$$

with end-point values $\Phi^+(0) = 1$ and $\Phi^+(\Delta t_Z) = 1/6$. This zoom-function is consistent with a behavior of the focal point of a system of two lenses the distance between which changes at constant rate. As $\Delta t_Z = 13.7$ s is known from inspection of the video clip, and we know that zoom-in starts at t_{ZT} , we find through least-squares procedure that $q_1 = 0.0461$ s⁻¹ and $q_2 = -6q_1 + 5/\Delta t_Z = 0.08833$ s⁻¹. We show the best-fit line in Fig. 4 together with data from Tbl. III, and find an excellent agreement.

For computational convenience we also introduce zoom-out function $\Phi^-(t)$ as,

$$\Phi^-(t) = \frac{1 + \lambda_1 t}{6 - \lambda_2 t}, \quad (3.10)$$

where we find $\lambda_1 = 0.1250$ s⁻¹ and $\lambda_2 = 0.2400$ s⁻¹.

In conclusion, once the zoom and tilt motions are known, it is straightforward to apply Eq. (3.1) to raw data, and to find the absolute elevation of the LM AS. This we report in Fig. 5 (black circles), and remark that this reconstruction is independent from the underlying motion that created it.

Time Start (s)	Time End (s)	Camera Motion	Tilt Rate ($^{\circ}/s$)	Description
-0.2	13.5	Zoom-Out		$\Phi^-(t + 0.2)$, Eq. (3.10)
13.5	27.2	Zoom-In		$\Phi^+(t - 13.5)$, Eq. (3.9)
2.1	20.0	Tilt Up	3	$\mu(t)$
20.0	21.9	No tilt	0	"
21.9	22.6	Tilt Down	-3	"
22.6	23.4	No tilt	0	"
23.4	24.4	Tilt Down	-3	"
24.4	25.1	No tilt	0	"
25.1	26.1	Tilt Down	-3	"
26.1	26.8	No tilt	0	"
26.8	27.8	Tilt Down	-3	"
27.8	28.2	No tilt	0	"

TABLE I: Transient behavior of the camera tilt and zoom as deduced from discontinuities in $\dot{\rho}$.

t (s)	$\mu(t)$ (deg)	Description
0.0	0.0	liftoff
2.1	0.0	tilt-up starts
20.0	53.7	tilt-up ends
21.9	53.7	tilt-down starts
22.6	51.6	tilt-down ends
23.4	51.6	tilt-up starts
24.4	48.6	tilt-down ends
25.1	48.6	tilt-up starts
26.1	45.6	tilt-down ends
26.8	45.6	tilt-up starts
27.8	42.6	tilt-down ends
30.0	42.6	

TABLE II: Linear interpolant table for the motion of the camera FOV mid-point, $\mu = \mu(t)$.

t^* (s)	$\Phi^+(t^*)$
20.0	0.44
21.9	0.36
22.6	0.31
23.4	0.28
24.4	0.25
25.1	0.25
26.1	0.20
26.8	0.17
27.8	0.15

TABLE III: Zoom values from discontinuity analysis. These are also plotted in Fig.4 together with their best-fit model for Φ^+ given in Eq. (3.9).

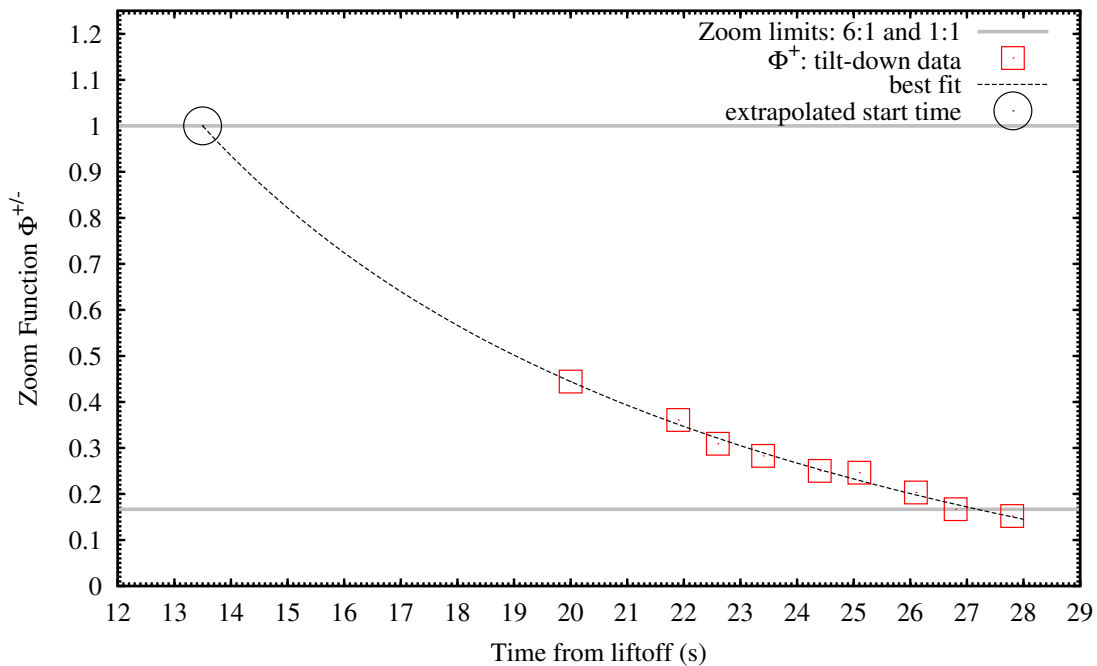


FIG. 4: Zoom values from Tbl.III and their best-fit model for Φ^+ , Eq. (3.9), are in excellent agreement with the zoom transition occurring at 13.5 s and lasting until 27.2 s after liftoff.

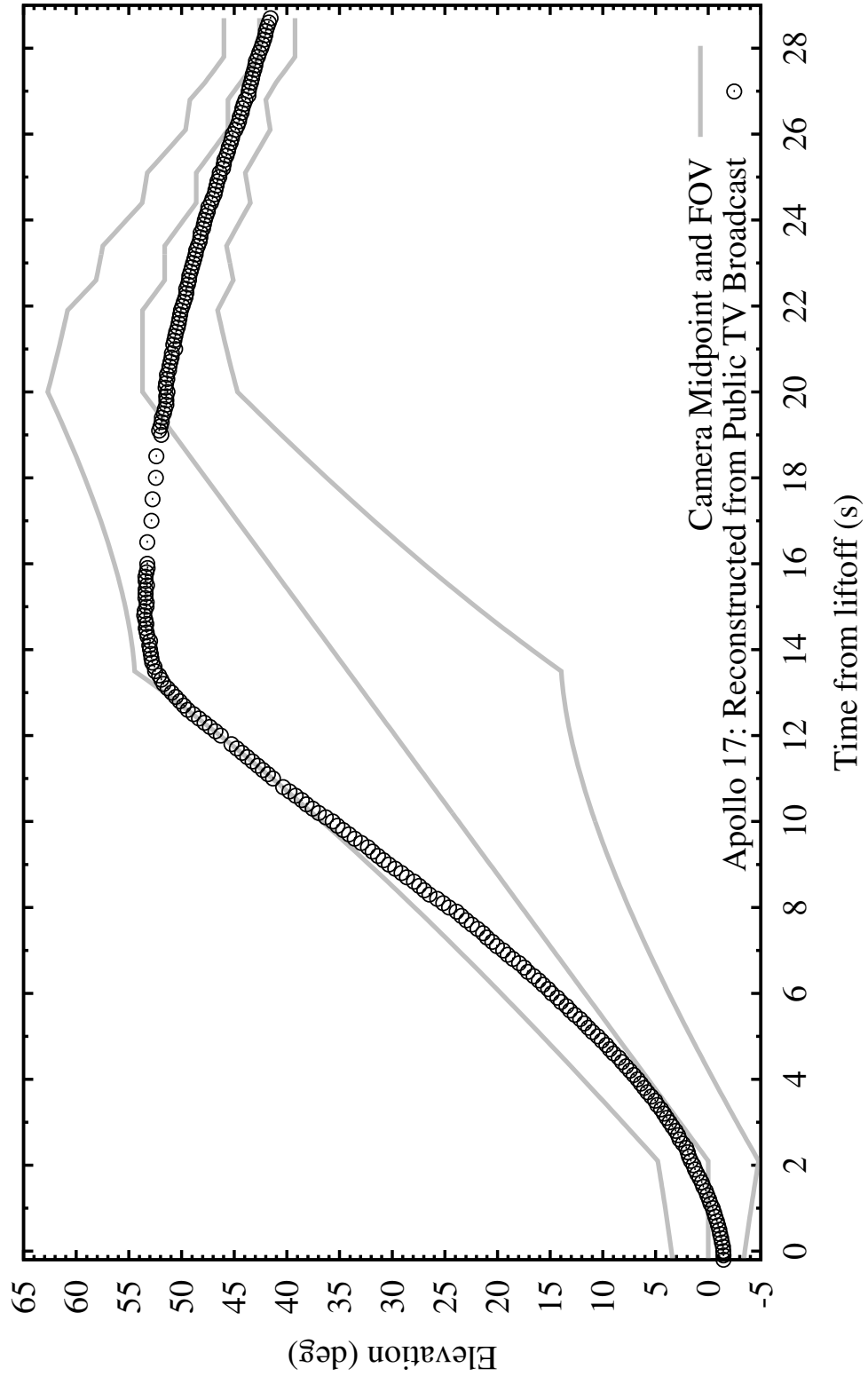


FIG. 5: Reconstructed elevation of the Apollo 17 LM AS ascent from the television broadcast [3] (black circles), and the camera tilt and zoom that captured it (gray lines).

4. APOLLO 17 VERTICAL RISE PHASE

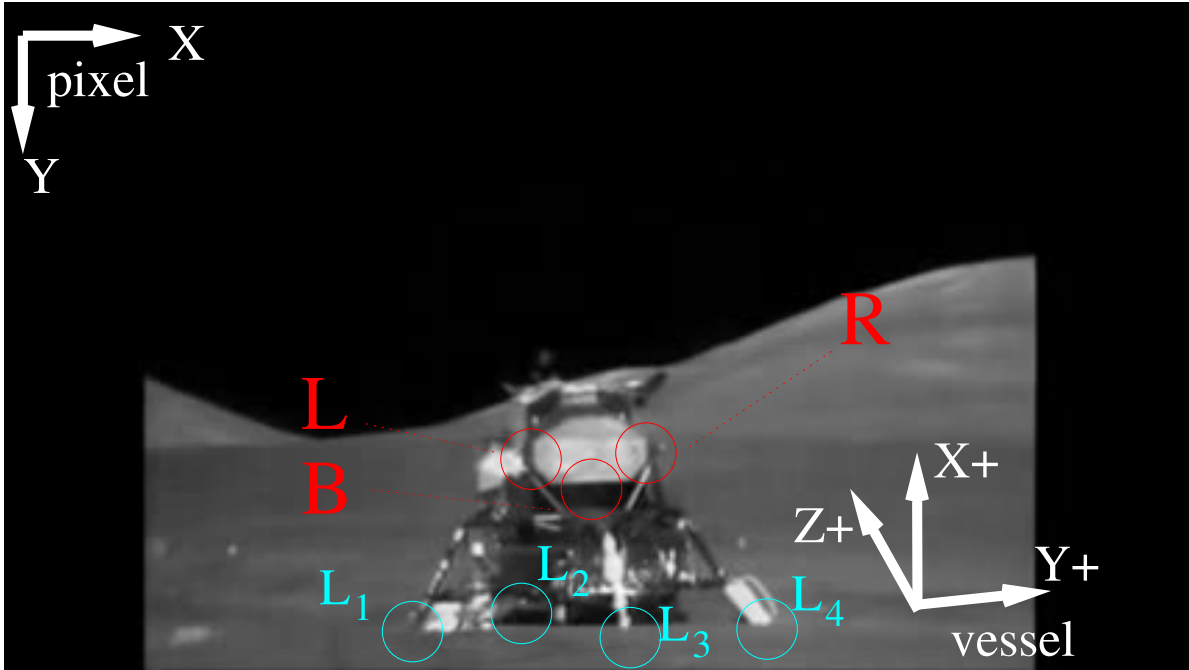


FIG. 6: The three points (L,R and B) on the crew compartment of the LM AS that were used to reconstruct the elevation, and the four landing gear feet ($L_{1..4}$) which positions were used to assess the scene angles. Also shown are the two coordinate systems used in assessing the motion.

The purpose of this section is to find the parameters of the constant acceleration motion during the Vertical Rise Phase, where we exclude the Descent Stage Clear. We first examine the scene, where we find the angle γ between the principal Z_+ axis of the LM AS and the line connecting the LM to the camera. Then, we use physical dimensions of the LM from [11] to determine how high is the camera above the launch site, and what is offset μ_{offs} between the horizon and the camera center. We combine this information with η from the previous section, and find thrust acceleration of the LM AS and its initial velocity.

The Scene

We assume that the landing surface is perfectly flat with respect to the camera, and that the camera vertical coincide with the landing surface vertical. Refer to the frame 1839₁₀ ([3] 03:03.9) in Fig. 6, from which we extract pixel positions of the landing gears.

As the front middle Z- landing gear is off-center from the midpoint between Y- (left) and Y+ (right), we find that the angle γ between the Z+ axis is

$$\gamma = 13.8^\circ, \quad (4.1)$$

to the left from the line connecting the camera with the LM center.

From the vertical angle between the feet Z- and Z+ we find that height of the camera above the launch site is

$$h_C = \frac{c^2}{\cos \gamma D} \cdot \Delta\phi = 7.2 \text{ m}, \quad (4.2)$$

where $c = 120 \text{ m}$ is the distance between the camera and the launch site, $D = 9.45 \text{ m}$ (=31 ft) is the diagonal distance between the feet, and $\Delta\phi = 0.25^\circ$ the vertical angle between two feet at 6:1 zoom.

Finally, from the height $h_{DS} = 3.2 \text{ m}$ of the Descent Stage (DS), and $h_{AS} = 2.8 \text{ m}$ of the AS, we find that the initial position of the point B on the LM is some

$$x_0 \approx -h_C + h_{DS} = -4 \text{ m}, \quad (4.3)$$

below the camera, which we take as a proxy for the LM AS position. As $x_0 = c \cdot \tan(\mu_{off} + \Phi^-(-0.2) \cdot \rho(-0.2))$, where $\rho(-0.2) = -1.45^\circ$, we conclude that the camera is initially pointed at,

$$\mu_{offs} = -0.45^\circ, \quad (4.4)$$

that is, below the horizon.

Motion

Inspection of the video clip suggests that starting from the frame 1941₁₀ ([3] 03:14.1) the Apollo 17 LM AS begins to change its pitch. This means that the Vertical Rise Phase terminates at $t_{VR} = 10.0 \text{ s}$ after liftoff. As the LM AS is moving straight up, the relationship between its height $x = x(t)$ and the elevation $\eta = \eta(t)$ is simply,

$$\tan \eta(t) = \frac{x(t)}{c}. \quad (4.5)$$

We have already found out the initial position of the LM AS with respect to the camera in Eq. (4.3). This is equivalent to $\eta(-0.2 \text{ s}) + \mu_{offs} = -1.45 - 0.45 = -1.9^\circ$. On the other hand, following the Descent Stage Clear, which ends at $t_{DSC} = 2 \text{ s}$ after liftoff, we expect

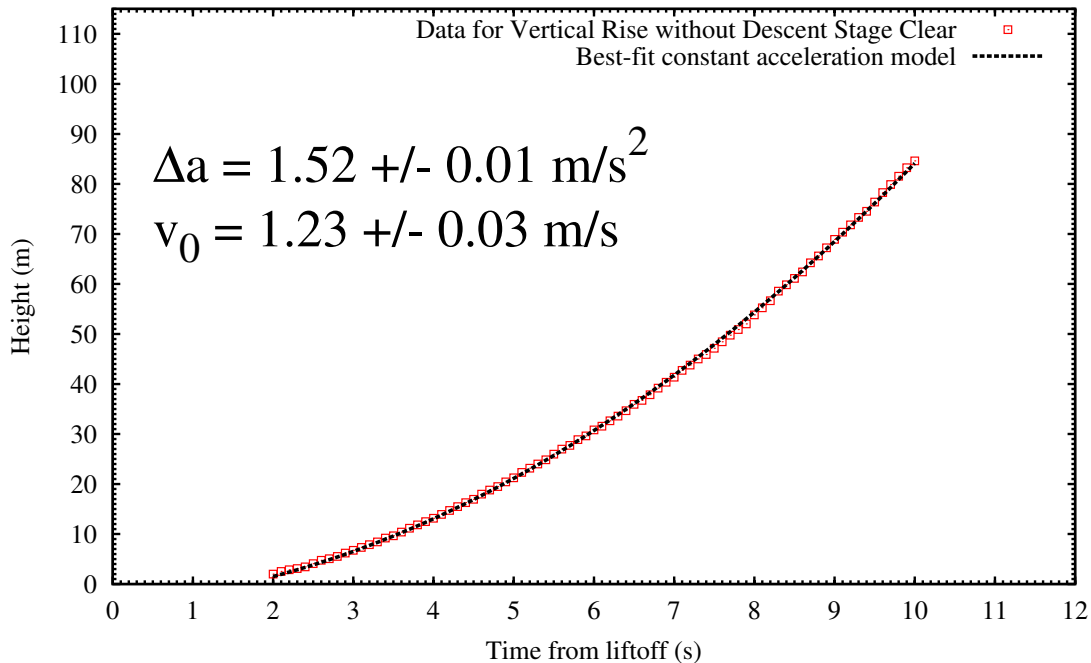


FIG. 7: Height of the LM AS as a function of time during the Vertical Rise Phase is consistent with the constant acceleration motion with the acceleration Δa and the initial velocity v_0 starting from x_0 from Eq. (4.6).

that in the time interval $t \in [t_{DSC}, t_{VR}]$ the motion of the LM AS is the constant acceleration motion,[12]

$$x(t) = \frac{\Delta a}{2} t^2 + v_0 t + x_0. \quad (4.6)$$

Here, v_0 describes the, so-called, “Fire-in-the-hole” (FITH), while $\Delta a = \tau_{A17} - g$ is the effective acceleration of the LM AS on the Moon with the gravity $g = 1.62 \text{ m/s}^2$. The FITH effect occurs when firing a rocket engine against close surface, and may lead to additional, albeit uncontrollable, thrust.

Using the least-squares procedure in which $x_0 = -4.0 \text{ m}$ is fixed, we find that during the Vertical Rise Phase the LM AS ascends with

$$\begin{aligned} \Delta a &= 1.52 + / - 0.01 \text{ m/s}^2, \\ v_0 &= 1.23 + / - 0.03 \text{ m/s}. \end{aligned} \quad (4.7)$$

In Fig. 7 we plot the elevations from the television broadcast [3] offset by μ_{offs} and the elevations from the best-fit constant acceleration model. As can be seen there, the agreement

between the two is excellent. The thrust acceleration of the Apollo 17 LM AS is thus

$$\tau_{A17} = 3.14 \text{ m/s}^2, \quad (4.8)$$

which is within 1% from the one reported in the Apollo 11 ascent, [4] $\tau_{A11} \simeq 3.19 + / - 0.03 \text{ m/s}^2$, and reproduced here in Fig. 2.

Discussion

The Apollo 17 LM AS thrust acceleration that we have extracted from the Vertical Rise Phase of the television broadcast [3], together with the initial velocity for which, we believe, the ‘‘Fire-in-the-Hole’’ phenomenon is responsible, fit the elevation data excellently. It also agrees with the information provided by NASA, [13, 14] and which we copy in Tbl. IV. This

Name	Quantity	Value [13, 14]	Value [15]	Value [4]
Lunar gravity (m/s^2)	g	1.622		
Ascent Stage height (m)	l_{AS}	3.76		
Descent Stage height (m)	l_{DS}	3.23 ^a		
Ascent Stage propellant mass (kg)	m_p^{AS}	2,359	2,376	
Ascent Stage total mass (kg)	m_0^{AS}	4,491 + crew	4,932	4,882
Time for Orbit Insertion (s)	t_{oi}			435
APS Thrust (N)	F_{th}	16,000		15,574
Propellant expelled velocity (m/s)	w	3,050	3,030	
Propellant mass flow rate (kg/s)	\dot{m}	5.25		5.14

^aThis assumes un-deployed primary struts.

TABLE IV: Lunar Module Data for Analysis of the Apollo 17 ascent.

information was base for the ascent analysis in [5], which proposed the value $\Delta a'_{A17}(\tau'_{A17}) \approx 1.49 (3.11) \text{ m/s}^2$.

As for the initial velocity v_0 for which the FITH phenomenon is deemed responsible, this is the first known estimate of its effects.

Overall we conclude that the Apollo 17 had approximately the same thrust accelerations as the Apollo 11 LM AS. For that reason in what follow we use interchangeable the Apollo 11 and the Apollo 17 data for the guidance logic and the nominal ascent trajectory.

5. NOMINAL EARLY ORBIT INSERTION

In this section we compute the elevations along what we call the “nominal” Orbit Insertion (OI) trajectory, which comprise of pitch adjustments identical to those reported for Apollo 11, [4] and which have already reproduced in Fig. 2. First notice that nominal $\Delta V_{nom} = 1,850$ m/s (=6073 fps), while the OI target for cross-directional velocity is $\dot{Z}_{oi} = 1,687$ m/s (= 5535 fps), meaning that the OI is done on tight fuel and time ($t_{oi} = 435$ s) budget.

What we assume by “nominal” is the following. We split the OI into two parts, the early and the late. The early comprises of pitch adjustments reported in [4] which are used until time t_{eeoi} . After that the late OI starts, $t \geq t_{eeoi}$, during which the pitch is continuously adjusted so that the radial acceleration is zero. It is not difficult to see that this strategy assures maximal cross-directional acceleration without loss of vertical velocity.

Consider the equations of motion of a rocket moving in XZ-plane in flat-Moon approximation (curvature of the Moon neglected for the duration of OI),

$$\ddot{x}(t) = \frac{\cos \theta(t) \cdot \tau_{A17}}{1 - \frac{\dot{m}}{m_0} t} - g \cdot \left(1 - \frac{\dot{z}^2}{\dot{Z}_{oi}^2} \right), \quad (5.1)$$

and,

$$\ddot{z}(t) = \frac{\sin \theta(t) \cdot \tau_{A17}}{1 - \frac{\dot{m}}{m_0} t} \quad (5.2)$$

for radial (height) and down-range (horizontal) positions, in that order. Here, $\dot{m} = 5.14$ kg/s and $m_0 = 4,882$ kg, while the term $\propto (\dot{z}/\dot{Z}_{oi})^2$ describes the centrifugal force. Per previous section the initial conditions are $z(0) = \dot{z}(0) = 0$, and $x(0) = -4$ m and $\dot{x}(0) = 1.22$ m/s. Solving Eqs. (5.1) and (5.2) suggests that the early OI ends at $t_{eeoi} = 90$ s, with $x_{eeoi} \approx 2.7$ km and $\dot{x}_{eeoi} \approx 46$ m/s, and $z_{eeoi} \approx 7.3$ km and $\dot{z}_{eeoi} \approx 191$ m/s. By the end of OI at $t_{oi} = 435$ s this trajectory reaches $\dot{z}_{oi} \approx 1,676$ m/s at distance $z_{oi} \approx 302$ km (= 163 n mi, cf. Fig. 15 in [4]), with $\dot{x}_{oi} \approx 46$ m/s and $x_{oi} \approx 18.3$ km. Overall, for the nominal trajectory the pitch adjustments $\theta = \theta(t)$ can be given either as a linear interpolant during early OI,

t (s)	$\theta(t)$ (deg)	Comment
0.0	0	Descent Stage Clear / Vertical Rise
10.0	0	“Pitch over”
16.0	52	
t_{eeoi}	52	end of early OI

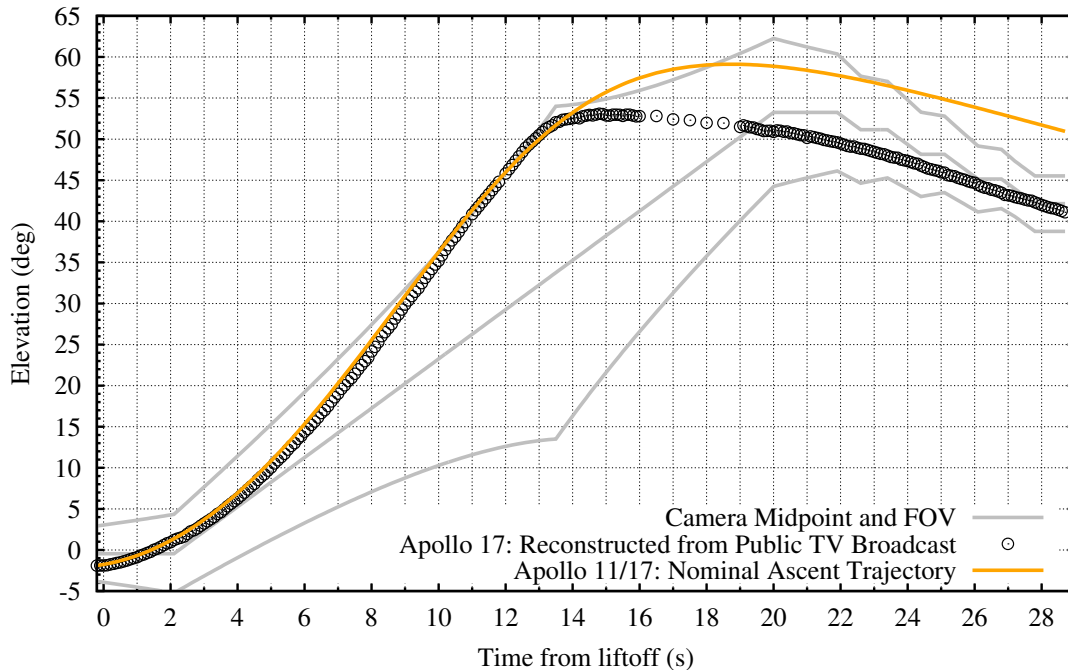


FIG. 8: Elevation of the Apollo 17 LM AS moving along the nominal ascent trajectory (in orange), and how it compares to the television broadcast [3] (black circles).

or as $\theta_{opt}(t)$ during later OI ($t \geq t_{eeoi}$), which is a solution of Eq. (5.2), $\ddot{x}(\theta_{opt}) = 0$.

The features of the nominal early OI are: (i), that the average pitch rate is

$$\dot{\theta}_{A11} = 9^\circ/\text{s}; \quad (5.3)$$

and (ii), that in the first $t_{eeoi} = 90$ s after liftoff the LM AS would continue to maintain non-zero radial acceleration.

Results and Discussion

The video clip shows no change of azimuth of the ascending LM AS, and so indicates that the XZ plane is already parallel to the CSM orbital plane. We thus find the LM AS elevation from,

$$\tan \eta(t) = \frac{x(t)}{\sqrt{c^2 + 2cz(t) \cos(\gamma) + z(t)^2}}, \quad (5.4)$$

where $c = 120$ m and $\gamma = 13.8^\circ$.

In Fig. 8 we show the nominal elevation (in orange) of the LM AS together with the broadcast elevation (black circles). As can be seen the two elevations seriously disagree for the entire early Orbit Insertion Phase in two important ways:

1. In time period 10 s to 14 s, the broadcast elevation is approximately equal to the nominal elevation, and the two start to depart from each other around 14 s. This suggests that the pitch-over starts at the pitch rate similar to or slightly slower than that of the Apollo 11 of $\dot{\theta}_{A11} \simeq 9^\circ/\text{s}$.

On the other hand, consider the following table of best fit values as function of duration of the Vertical Rise Phase t_{VR} ,

t_{VR} (s)	Δa (m/s ²)	v_0 (m/s)	Comment
10.0	1.52	1.23	actual
14.0	1.55	1.23*	extended

where * means that the value was fixed to the best fit value v_0 found for $t_{VR} = 10$ s. What this table tells us is that by artificially extending the duration of the Vertical Rise, the vessel (vertical) acceleration appears to be increasing. However, were we following a rocket, with the pitch-over the acceleration would be decreasing because the rocket engine would no longer be pointed to the ground. Consider that given propellant mass flow rate from Tbl.IV, at $t'_{VR} = 14$ s the momentary vessel mass (acceleration) would have decreased (increased) by 1.5%, meaning that the average acceleration over the entire interval could not increase by 2%.

In other words, the acceleration of the vessel in the broadcast appears to be increasing with increasing pitch angle, which non-throttleable rockets cannot do.

2. At 14 s after liftoff, the vessel in the broadcast makes a sharp downward turn that requires a higher turning rate than that of Apollo.

We conclude that the broadcast trajectory either features an LM AS which guidance logic decided to perform non-optimal pitch adjustments, or does not feature an LM AS at all. We examine both of these conclusions in the next section.

Next, we go back to Mr. Fendell's effort to track the LM AS during the ascent. First notice that the Tbl.II suggests that four camera tilt-downs he executed can be divided in two groups, the first tilt-down of duration 0.7 s comprises the first group, while the other three tilt-downs of duration 1 s each and mutually separated by 0.7 s comprise the second group.

Capturing the nominal trajectory in Fig. 8 (orange line) by the camera requires only the

first tilt-down, meaning that the second group of tilt-downs is unnecessary.

This coincidence leads us to hypothesize that Mr. Fendell anticipated that the LM AS would be ascending along the nominal Apollo 11-like trajectory and planned only the first tilt-down of the camera. He believed this would give him ample time later to command the camera to pan to the left. This sequence of camera motions would have maintained the LM AS in sight for a very long time before it would have eventually disappeared beyond the horizon.

Maybe what happened instead was the following: To his surprise, Mr. Fendell noticed that after the “pitch-over” the LM AS started to move down much faster than he anticipated. In an attempt to continue tracking the LM AS, he started the second group of tilt downs. While doing these he was either too busy, or did not have the communication bandwidth, to command the camera to pan to the left. As a result, the camera lost track of the LM AS as it left the camera FOV on the left side. We speculate that to this day Mr. Fendell wonders what happened that day and why his estimate of the ascent trajectory was so off that he lost the LM AS some 29 s after liftoff, when he thought he had the entire early Orbit Insertion covered.

6. ALTERNATIVE EARLY ORBIT INSERTION TRAJECTORIES

Following the conclusion of the previous section that the origin of the vessel featured in the television broadcast [3] is uncertain, we explore two alternative hypotheses in regard to the vessel origin.

The first vessel we examine is, what we call, a “space-rocket,” which features a hypothetical LM AS carrying the same Ascent Propulsion System (APS) but with the Reactive Control System (RCS) with greater turning rate than that of Apollo.

The second vessel, which we refer to as a “roller coaster,” is not a space-vessel at all. We consider it because it in an interesting way resolves all of the inconsistencies in the reconstruction of the broadcast elevation mentioned so far.

6.1. Space-Rocket

The broadcast suggests that the vessel featured in it had the greater turning rate. The video clip suggests that the pitch-over starts at 10 s and the frame 1941₁₀ ([3] 03:14.1). By the time 11.5 s and the frame 1956₁₀ ([3] 03:15.6), as the LM AS is turning, we have direct view of its bottom. At that time its elevation is $\eta(11.5 \text{ s}) \approx 45^\circ$, so the pitch is

$$\theta(11.5 \text{ s}) = 90^\circ - \eta(11.5 \text{ s}) \approx 45^\circ. \quad (6.1)$$

Thus, the pitch rate $\dot{\theta}_{max}$ for completion of this turn is,

$$\dot{\theta}_{max} \approx \frac{45^\circ}{1.5 \text{ s}} = 30^\circ/\text{s}, \quad (6.2)$$

which is considerably higher than the turning rate of $9^\circ/\text{s}$ the Apollo 11 and 17 were reported having.

This said, we describe the early Orbit Insertion (OI) trajectory through the pitch angle $\theta = \theta(t)$, which is a linear interpolant over a table of values $\Theta = [t_i, \theta_{t_i}]_{i=1,20}$, given by

$$\Theta = \begin{bmatrix} 0 & 0 \text{ (fixed)} \\ 10 & 0 \text{ (fixed)} \\ 11 & \theta_{11} \\ 12 & \theta_{12} \\ \cdot & \cdot \\ 28 & \theta_{28} \end{bmatrix}. \quad (6.3)$$

We introduce the quantity $\vec{w} = \{w_i\}_{i=11,28}$, which we use to represent the pitch changes as,

$$\delta\theta_i = \frac{\dot{\theta}_{max} \cdot \Delta t}{\cosh(w_i)}, \quad (6.4)$$

where $\Delta t = 1 \text{ s}$, and

$$\theta_i = \min(\theta_{MAX}, \sum_{k=0}^i \delta\theta_k), \quad (6.5)$$

with θ_{MAX} being the pitch that the LM AS will maintain for the remainder of the early OI. This parameterization guarantees that the pitch adjustments θ_i are non-decreasing, and that between two consecutive Δt -steps, the pitch angle does not change more than the maximum $\dot{\theta}_{max} \cdot \Delta t$. The optimization procedure is performed with respect to \vec{w} , while the

function being minimized is

$$\text{MSRE}^2(\vec{w}) = \sum_i \left(\tan \eta_i - \frac{\hat{x}(t_i; \theta(\vec{w}))}{c + \cos \gamma \hat{z}(t_i; \theta(\vec{w}))} \right)^2. \quad (6.6)$$

We perform two optimizations, one for each $\theta_{MAX} = 52^\circ, 59^\circ$.

Results and Discussion

In Fig. 9 (top panel) we show two most optimal space-rocket trajectories (in red for $\theta_{MAX} = 52^\circ$, in brown for $\theta_{MAX} = 59^\circ$) and their pitches as the functions of time (bottom panel, same color scheme). From there it can be seen how they compare to the nominal trajectory from the previous section and the broadcast trajectory.

Firstly, the space-rocket with the turn constrained to $\theta_{MAX} = 52^\circ$ fails in the same way as the nominal trajectory: They both start to turn too quickly, but then they stop turning and so overshoot the broadcast trajectory.

The space-rocket with $\theta_{MAX} = 59^\circ$ succeeds in capturing the most of the latter part of the broadcast trajectory, so it could be called a “best-fit” trajectory. However, like the other rocket trajectories considered so far, it starts to turn too quickly, and so fails to describe the transition from Vertical Rise to the Orbit Insertion occurring between 12 s and 18 s into the flight. Then, given the discussion of the nominal trajectory from the previous section, now the early OI appears to be ending at $t_{eeoi} = 12.2$ s. If the vessel continues like this for the remaining of the OI, it won’t be able to reach OI target height, but will have horizontal velocity that exceeds the OI target. Such an OI thus requires the third corrective stage, which starts when the horizontal target velocity is reached. At that time the vessel turns to vertical and continues that way until it reaches the OI target height, at which point its pitch returns to horizontal. Overall, such an 59° -rocket-trajectory becomes quite complicated to execute: it puts enormous computational strain on the guidance logic to compute it, while requiring additional maneuvering unlike anything else done so far.

In conclusion, neither the guidance logic nor the Apollo RCS are capable to perform the OI along the “best-fit” 59° -rocket-trajectory. In addition, neither this trajectory supports the turning schedule seen in the broadcast (at 11.5 s the LM AS is pitched at 30° and not at 45°).

As we show next, it is possible that the featured vessel is not even a rocket.

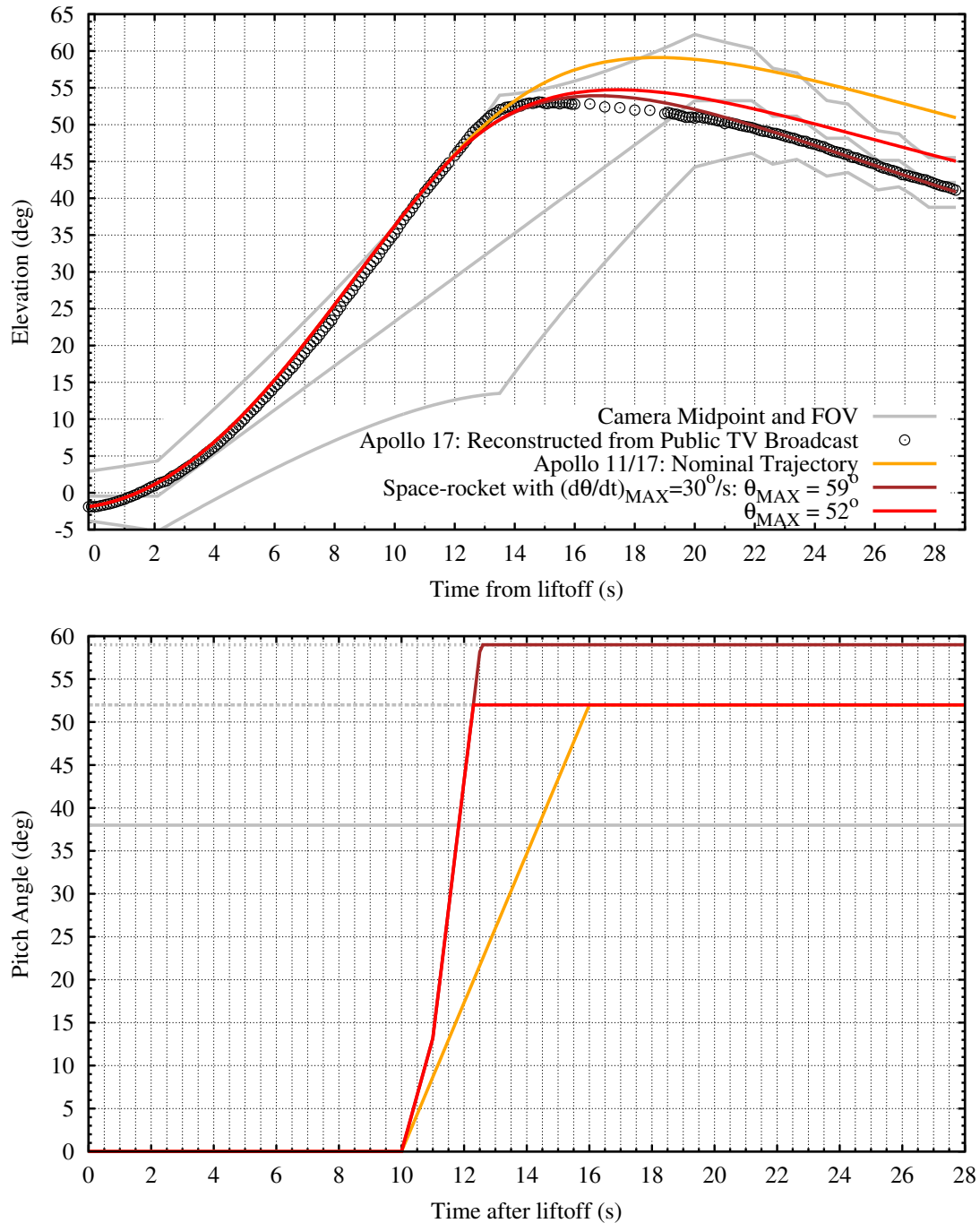


FIG. 9: (Top panel) Best-fit elevations of space-rocket with $\theta_{MAX} = 59^\circ$ (in brown) and $\theta_{MAX} = 52^\circ$ (in red), nominal from the previous section (in orange), all compared to the broadcast elevation (black circles). (Bottom panel) Pitch adjustments for each trajectory. The best-fit trajectory with $\theta_{MAX} = 59^\circ$ (in brown) requires quicker RCS than that of Apollo, while it results in an Orbit Insertion that the Apollo's guidance logic cannot execute.

6.2. Roller Coaster

We summarize failures of so far discussed vessels, in reproducing the broadcast:

- The space rockets fail to describe the transition between the vertical rise and early orbit insertion elevations in that they turn too quickly. Best-fit rocket manages to capture some of the early orbit insertion.
- Following the completion of the Vertical Rise Phase, it appears that by turning away from the vertical, the acceleration of the vessel is increasing. This behavior is inconsistent with the space rocket, which acceleration decreases as the thrust is pitched away from the vertical.
- Toward the end of tracking at $t^* \simeq 28.5$ s, the distance between the vessel and the camera is much greater than the nominal trajectory would allow.

This we estimate as follows. After some 16 s of flight the vessel appears to be morphing into a cone. This we interpret as the tip of the cone being the vessel and the body of the cone being the exhaust plume from its engine. This on its own is unusual considering that the exhaust plume was present during the first 0.8 s after liftoff, but then disappeared.

At that time the size of the tip of the cone is $\Delta P_L \simeq 4$ pixels horizontally, corresponding to the angle $\Delta\alpha = \Delta P_L \cdot \text{HFOV}(6:1)/\text{HFRAMESIZE} = 0.11^\circ$. The maximal width of the LM AS is $L \simeq 4.3$ m (=14.1 ft), so the distance d to the vessel is

$$d \sim \frac{L}{\Delta\alpha} = 2.2 \text{ km.} \quad (6.7)$$

On the other hand, along the LM AS nominal trajectory from Sec. 5, at t^* we find for the position $x^* \approx 540$ m, $z^* \approx 311$ m, and velocity $\dot{x}^* \approx 28$ m/s, and $\dot{z}^* \approx 40$ m/s, so the distance is $d^* \sim 0.7$ km.

Why a Roller Coaster?

As we show next, all of the listed features are consistent with a “roller coaster.” The roller coaster pertains to a λ -scaled down model of the LM AS, which is sliding along tracks. While the entire scene is staged in scaled down form, it is represented as being full-scale, that is, scaled-up by $1/\lambda$. NASA documentation suggests $\lambda = 1/10$. [16]

One way the Vertical Rise Phase can be simulated is for the LM AS scaled-down model to be propelled by a jet engine, which full-scale thrust acceleration is $\tau = \Delta a + \frac{g_E}{\lambda}$, where $g_E = 9.82 \text{ m/s}^2$ is the Earth gravity. Consider sliding along the tracks that are at time t pitched at an angle $\theta(t)$ with respect to the vertical, and let $l = l(t)$ describe full-scale position of the model along the tracks. The full-scale equations of motion then reduce to,

$$\ddot{l} = \Delta a + (1 - \cos \theta(t)) \cdot \frac{g_E}{\lambda}, \quad (6.8)$$

with,

$$\begin{aligned} \dot{x}(t) &= \cos \theta(t) \cdot \dot{l}(t), \\ \dot{z}(t) &= \sin \theta(t) \cdot \dot{l}(t). \end{aligned} \quad (6.9)$$

Here, $\Delta a = \tau_{A17} - g$, as before, while the full-scale initial conditions are $x(0) = -4 \text{ m}$ and $z(0) = 0$, with $l(0) = 0$ and $\dot{l}(0) = \dot{x}(0) = 1.22 \text{ m/s}$.

From Eqs. (6.8) and (6.9) it is obvious that for constant τ the vertical acceleration of a roller coaster increases as the tracks it is sliding along, change their pitch away from the vertical. It is not difficult to see that the vertical acceleration is maximal at 60° pitch. On the other hand, because $\tau \approx g_E/\lambda \gg \Delta a$, with pitch-over the horizontal acceleration will be huge, so the distance from the roller coaster to the camera will start to increase at a rate considerably higher than the distance from an LM AS flying along the nominal trajectory. In fact, it is not difficult to see that the size of the scaled-down trajectory of the roller coaster will be approximately equal to the size of the full-scale nominal trajectory.

With this in mind, we construct the pitch-adjustment function $\theta = \theta(t)$ as a linear interpolant over a table of values $\Theta = [t_i, \theta_{t_i}]_{i=1,20}$. This time we simplify the pitch increments as,

$$\Theta = \begin{bmatrix} 0 & 0 \text{ (fixed)} \\ 10 & 0 \text{ (fixed)} \\ 11 & |\delta\theta_{11}| \\ 12 & |\delta\theta_{11}| + |\delta\theta_{12}| \\ \cdot & \cdot \\ 28 & \sum_{i=11}^{28} |\delta\theta_i| \end{bmatrix}, \quad (6.10)$$

which assures us that the pitch angles $\theta_{t_i} = \sum_{k=11}^{t_i} |\delta\theta_k|$ are strictly increasing. Unlike the space-rocket trajectories, here we do not limit neither $\delta\theta_{t_i}$ nor θ_{MAX} .

Results and Discussion

In Fig. 10 we show the best-fit roller coaster trajectory (top panel, in blue) and the pitch adjustments it requires (bottom panel), and how they compare to the nominal trajectory from the previous section (in orange) and to the broadcast trajectory (top panel, black circles). As can be seen the agreement between the roller coaster and the broadcast trajectory is excellent. Then, through fine pitch adjustments the roller coaster can capture minute changes in the broadcast elevation, so well, that any further analysis would require more precise broadcast elevations than what we provided in this report.

For that reason the following discussion is qualitative. Two observations from the analysis of the broadcast that require additional explanations are the distance to the camera, which at $t^* = 28.5$ secs should be around 2.2 km, and the pitch, which at $t^{**} = 11.5$ s should be 45° .

From solving the roller coaster equations, we know that the full-scale distance of the roller coaster at t^* is $d^* \sim 3.3$ km, while the pitch is $\theta^{**} \simeq 3^\circ$ at t^{**} .

It is not difficult to see that the LM AS model propelled by a throttleable jet engine can explain the two observations. This would modify the optimization where we would add two additional constraints (distance d^* and pitch θ^{**}), on one hand side. On the other, we would have to find a convenient way of parameterizing the jet engine thrust as a function of time. One such trajectory could be where the jet engine is turned off once the model enters the curved tracks (at 10 s), and then turned back on around 16 s, possibly with the afterburner. Here we intentionally mention the afterburner: It could be an indication that the jet engine is again operating, while its huge exhaust plume makes it visible even if the vessel is far away from the camera.

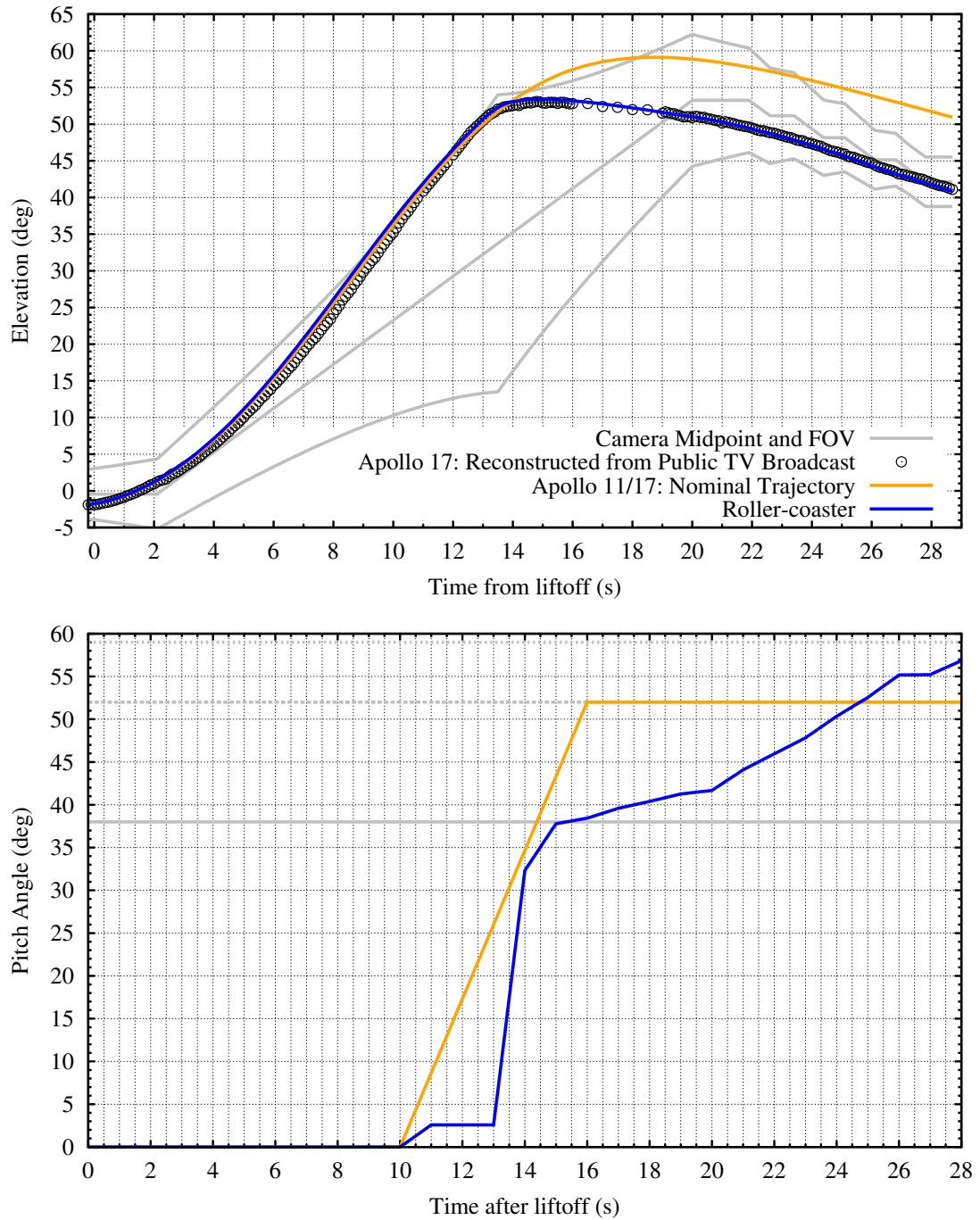


FIG. 10: The roller coaster trajectory (top panel, in blue) with the pitch adjustments (bottom panel) reproduces the elevations from the broadcast (top panel, black points) almost perfectly. For comparison we show the elevation along the nominal trajectory (in orange, top panel), and its pitch adjustments (bottom panel)

7. CONCLUSIONS

In this report we have reconstructed the elevation of the ascending Apollo 17 Lunar Module Ascent Stage from the vantage point of the camera that was transmitting the ascent. In carrying out the reconstruction we, in effect, have recovered the steps that NASA camera operator Ed Fendell performed in order to track the ascending LM AS with the camera. From Mr. Fendell's actions we have concluded that he was surprised by the way the LM AS was ascending.

From the analysis of the Vertical Rise Phase, and using the reported Apollo 11/17 pitch adjustments we have constructed, what we call, the “nominal” ascent trajectory, and found that it considerably overshoots the trajectory from the broadcast. This motivated us to investigate two alternatives:

The first was the “space-rocket,” which featured a Reactive Control System (RCS) that has considerably higher turn rates than the Apollo RCS. One such 59° -trajectory fitted the latter part of the broadcast, but failed in the following aspects: *(i)*, it did not reproduce the transition between the Vertical Rise and the early Orbit Insertion; *(ii)*, flying along such trajectory could not be performed with the Apollo guidance logic; and *(iii)*, its pitch adjustments seriously disagreed with the ones actually reported.

The second was the “roller coaster,” featuring a scaled-down scene in which a model of the LM AS was moving along tracks that mimicked the ascent trajectory. We were able to reconstruct tracks (their curvature, or pitch), motion along which reproduced the broadcast (almost too) perfectly. This suggested that the approach is capable of explaining even the minute variations of motion in the broadcast.

From the failure of space rocket trajectories in describing the Apollo 17 broadcast we conclude that the broadcast does not feature the LM AS lifting off the lunar surface.

Conversely, from the success of the roller coaster in description of the broadcast we therefore conclude that the broadcast shows an ascent that has been staged in a specialized film studio on Earth.

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